

ADVANCED GEOMETRIC METROLOGY

MASANAO MORIMURA

NATIONAL RESEARCH LABORATORY OF METROLOGY

SAKURA-MURA, NIIHARI-GUN

IBARAKI 305, JAPAN

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OUTLINE

1.	Introduction	137
2.	Precision Measurement of Length and Angle	137
2.1	Use of Natural Periodicity	
2.1.1	Use of Interference Phenomena	
2.1.2	Interferometers Used for Geometric Metrology	
(a)	Michelson interferometer	
(b)	Fabry-Perot interferometer	
(c)	Fizeau interferometer	
(d)	Angle measuring interferometer	
2.2	Use of Artificial Periodicity	
2.2.1	Artificial scales	
(a)	Diffraction grating	
(b)	Magnetic scale	
(c)	Electromagnetic scale	
2.2.2	Photo-electric optical level	
(a)	Jones' optical lever	
(b)	Use of moiré fringes	
3.	Discrimination of The Direction of Translation	142
4.	Techniques for Maintaining Accuracy and Precision Relevant to Geometric Metrology	143
4.1	Fine mechanisms for linear translation and rotation	
4.2	Compensation for the Incomplete Translation	
4.3	Multiplication of the Optical Path	
4.4	Use of Modulation Techniques	
4.4.1	Amplitude modulation	
4.4.2	Phase/frequency Modulation	
5.	Elimination of Environmental Disturbances	146
5.1	Vibration isolation	
5.1.1	The seismo-system of one degree of freedom	
5.1.2	Vibration isolator	
5.1.3	Isolation against sound wave disturbance	
5.2	Temperature Control	
5.3	Compensation for environmental disturbances	
6.	Acknowledgement	148
	References	149

ADVANCED GEOMETRIC METROLOGY

Masanao Morimura

National Research Laboratory of Metrology, Japan

1. Introduction

Precision measurement of length and angle, or geometric quantities, is itself basic and also applied to the precision measurement of various physical quantities, since measurement of many physical quantities are reduced to the measurement of the geometric quantities. For example, when the mercury column manometer is used for the absolute measurement of pressure, the precision measurement of the mercury head is indispensable. The inductance of a cylindrical coil is determined by measuring the diameters of the cylinder and the wire and the pitch of the winding, and the capacitance of a parallel plate capacitor is determined by measuring the distance between two electrodes and the surface area of the electrode.

The geometric metrology covers the methods of measuring length and angle, the implementation of the methods, the establishment of the methods of calibration, techniques to improve the accuracy and precision of measurement as well as the evaluation, and techniques to eliminate or reduce the influence of the environmental disturbances.

The essentials are summarized in this paper.

2. Precision Measurement of Length and Angle

Since length can be measured by comparing the magnitude to be measured with the reference scale, it is essential to build up the accurate reference as well as the detecting technique. The reference scale must have periodic structure in space, and the period must appropriately be chosen according to the measurement requirement.

At the length measurement using ordinary scale, the graduation lines are easily observed and also interpolated by eyes. As the scale interval or the period of the periodic structure gets smaller, it gets indispensable to use a detector of the "graduated lines" sensitive enough to discriminate them, or to devise to enlarge the "scale interval".

The periodic structure is constructed either utilizing natural phenomena or artificially. Accuracy of the length measurement is principally limited by the accuracy of the period.

Self calibration is possible for the angle measurement if it is utilized that the whole angle of a circle is 2π , and the accuracy of the angle measurement is limited by the accuracy of the division of a circle.

Angle can also be measured by taking the ratio of the lengths of two sides of a right triangle. Then angle measurement is reduced to length measurement.

Length is also measured by measuring the time of flight of electromagnetic pulses, but the accuracy is not high in laboratory

measurement with the present technology.

2.1 Use of Natural Periodicity

Although any periodic phenomenon can be utilized for the reference scale, electromagnetic or sound wave are conveniently used. It is well known that these waves propagate with a constant speed c in uniform medium, and the wavelength λ and the frequency ν are related to c as

$$\lambda\nu = c. \quad (1)$$

Therefore the wavelength is constant when the frequency is fixed, and the amplitude of the wave changes periodically and sinusoidally in ideal case with constant wavelength.

Optical wave is conveniently used since many kinds of techniques have been developed in this wavelength region. Although the wavelength of X-ray or γ -ray is as small as the order of sub nm, the technique has not been developed to utilize these radiation to the reference scale for the measurement of geometric quantity. Lattice spacing of a perfect single crystal, which is of the order of 0.1nm, may be utilized in near future.

Ultrasonic wave of microwave frequency region can be generated on the surface of a crystal or thin film and is recently utilized for the microscope.

2.1.1 Use of Interference Phenomena

In order to build up a reference scale by the wave phenomena, it is essential to utilize the interference of two waves. When two waves are superposed, the resultant amplitude varies from the maximum to the minimum as the phase difference varies from $2n\pi$ to $(2n+1)\pi$, n being integer. There is no way to detect the wavelength of light directly but the interference fringes since the latter is stationary in space however the wave is propagating, and also enlarges the "scale interval". Interval of the adjacent maximum (or minimum) of the interference pattern corresponds to $\lambda/2n$ and constant in homogenous media, therefore length or displacement can be measured in units of $\lambda/2n$ by counting the number of the interference fringes. The visibility V of interference fringe is an index to evaluate the quality of interference, and is defined as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}, \quad (2)$$

where I_{\max} and I_{\min} is the maximum and the minimum of the intensity of the interference fringes as shown in Fig.1.

V is usually less than unity which is the maximum in the ideal case. It is noted that the interval of the interference fringes is independent of V . This is one of the feature of interference fringe as utilized to the reference scale. For the measurement of length or displacement smaller than the fringe interval, it must be interpolated. Analog or digital techniques have been developed for the purpose as stated later. The precision of interpolation is limited by the distortion of the waveform, since interpolation is done by measuring the phase under the assumption of pure sinusoidal change of

the intensity. It is to be noted that we are measuring the optical phase ϕ of the interference fringes which is

$$\phi = kd, \quad (3)$$

where k is the wave number ($=2\pi/\lambda$) and d is the geometrical distance, respectively, and λ is the wavelength in the medium. When we measure length or displacement in air, change of the wavelength due to the change of refractive index must be taken account if the

accuracy of 10^{-6} or better is required. It is desirable to have wave source of good monochromacy. The measurable length is closely related to the spectral purity of the wave source.

2.1.2 Interferometers Used for Geometric Metrology

Two types of interferometer are currently used for the precision measurement of length: the Michelson interferometer and the Fabry-Perot interferometer. The Fizeau interferometer is used for small angle measurement.

(a) Michelson interferometer

The Michelson interferometer is a two beam type: light beam from the source is split into two plane waves by a beam splitter and recombined there after reflected by each flat mirror. Fig. 2 shows the scheme. In this configuration, length or displacement is measured in units of $\lambda/2$. Modifying the configuration, it is possible to make the unit smaller. When, as is shown in Fig.3, the mirror M_1 in Fig.2 is replaced by a cube-corner prism P and a totally reflecting mirror is fixed in front of P so that the beam exit from M_1 may come back, the optical path length gets twice as long compared with the configuration shown in Fig.2, and the "scale interval of $\lambda/4$ is obtained.

Various variations have been designed some of which will be shown later.

Sinusoidal interference fringes are obtained with the Michelson interferometer, which is convenient to interpolate fringes. Either digital or analog interpolation is possible electronically. Analog interpolation is superior: as fine as $1/3000$ of fringes can be interpolated with the elaborate technique.

(b) Fabry-Perot interferometer

The Fabry-Perot interferometer is a multiple beam type: light beam from the source enters in two parallel plate glass with inner coating of very high reflectivity and exits after reflected between the plates many times. It results in very narrow fringes and is utilized as the scale of $\lambda/2$ steps, but it is inconvenient for interpolation.

Fig.4 shows the outline of combined X-ray and optical interferometer used for the measurement of the repeat distance of the Si lattice where the Fabry-Perot interferometer is used for providing the scale of $\lambda/2$ steps. Plane and spherical mirrors are used there.

The profile of the interference fringe changes sensitively with the tilt of the mirrors, and the asymmetry appears when the parallelism is incomplete. This leads to a systematic error in the precision length measurement.

(c) Fizeau interferometer

The Fizeau interferometer is also the multiple beam type: it consists of a reflecting mirror and a semi-transparent mirror disposed closely and inclined slightly as shown in Fig.5.

When a light beam is introduced to the interferometer through a semi-transparent mirror, equidistance interference fringes are observed and the interval of the adjacent fringes R depends on the angle between two mirrors. Denoting it β , R is given as

$$R = \frac{\lambda}{2\beta}, \quad (4)$$

where λ is the wavelength of the source.

(d) angle measuring interferometer

The Fizeau interferometer is modified to attain as twice or four times as high sensitivity. Fig.6 is an example.²⁾ It is not only 4 times sensitive but also free from environmental disturbances since the optical paths are the same for the beams travelling in the opposite direction and the optical path difference is approximately zero.

The measuring range of the Fizeau interferometer is rather narrow and the another type of interferometer has been studied for angle measurement. The schematic diagram is shown in Fig.7. Two cube-corner prisms put on a rotary table RT with distance D_c reflect collimated light beams from the source. The reflected beams are recombined by the beam splitter BS and impinge on the detector. The optical path difference d caused by the rotation of RT is

$$d = D_c \sin \theta - D_0, \quad (5)$$

where D_0 is a constant corresponding to the optical distance of BS and M, and the light intensity I at the detector is

$$I = I_0 \{1 - a \cos(2\pi x)\} \quad (6)$$

where $x = 2d/\lambda$, and, I_0 and a are the instrumental constants.

Determining x by measuring the intensity change, the angle of rotation is determined if D_c and D_0 are previously calibrated. This interferometer can be modified to have as twice as high sensitivity by using another reflecting mirror as shown in Fig.8, and the measuring

range of 15° with the accuracy of $0.01''$ has been obtained.³⁾

2.2 Use of Artificial Periodicity

Graduated lines engraved on a flat glass or metal plate has long been used as a scale for length measurement. It is not sufficient for precision measurement since the scale interval cannot be made smaller than 0.1mm and interpolation is not possible. Various devices substituting it have been invented to provide the reference scale

which can be used for precision measurement.

Although both the resolution and the accuracy are usually poorer than the natural periodicity, they are easily handled and the scale interval can be made round number in units of the meter.

2.2.1 Artificial Scales

(a) Diffraction grating

Diffraction grating used for spectroscopy is utilized for length measurement. The scale interval can be made as small as $1\mu\text{m}$. Two gratings of the same pitch is used; one is fixed and the other is translated in parallel. When these gratings are illuminated, the intensity of the transmitted light varies with the same period as the pitch, and length or displacement is measured by counting the period. If the direction of the graduations is not parallel to each other but tilted slightly, moiré pattern is observed as shown in Fig.9. The pattern moves perpendicular to the direction of translation of the grating, and the pitch is approximately equal to d/θ when the angle θ is small. If the higher order diffracted ray is cut out, the intensity distribution of the moiré pattern gets sinusoidal even though a binary grating is used, then the interpolation is made possible.

The pitch errors of the grating is averaged out since many grating lines are illuminated at once by the collimated beam.

(b) Magnetic scale

Suppose a signal of constant frequency is recorded on a magnetic tape moving in constant speed v , then the remanence on the surface of the magnetic tape changes periodically with the pitch $p = v/f$. This is the principle of building up the reference scale by the magnetic scale. In order to detect the remanence, the flux-sensitive head is used where the technique of magnetic modulation is applied. Although the pitch cannot be made small as the optical grating, very fine interpolation is possible using phase modulation technique as stated later and the resolution of $1\mu\text{m}$ can be obtained. Magnetic scale can be made as long as 2m which is very difficult for the optical grating.

(c) electromagnetic scale

Rectangular pattern as shown in Fig.10 evaporated on an insulating substrate can be used as the reference scale. When a.c. current flows in the pattern, a.c. magnetic field is generated around it and is detected by the replica. Since the sinusoidal signal is obtained, it is easy to interpolate the pitch and the resolution of $1\mu\text{m}$ is attained. The measuring range is not so large as the magnetic scale.

2.2.2. photo-electric optical lever

(a) Jones' optical lever

Fig.11 shows the schematic plan of a photo-electric optical lever designed by Jones et al.⁴⁾ The image of the grid G_1 , appears in the plane of G_2 which is the replica of G_1 but is split into two halves, separated by the width of a bar in the grids, so that when the right half of G_2 passes the light from G_1 , the left half of G_2 stops the light from G_1 . Viewed from L_2 , the effect is that of a vertically divided field, in which the halves go alternately black and white in

anti-phase as the mirror M is rotated. A narrow angled prism P is placed behind one half of L_2 with its vertex horizontal to divert the light from this half downwards to fall as a focused image of source S on the lower half X_2 of the photocell, while the upper half is undiverted. As M rotates, both images maintain their positions but their intensities change, due to the varying degrees of overlap between the image of G_1 and the two halves of G_2 . It can detect a change of 0.1 nrad in orientation of a 2mm mirror, with a response time of about 0.25s. The system has been widely applied to various precision measurements.

(b) use of moiré fringes

Fig.12 shows the schematic plan of the small-angle measuring

device utilizing moiré fringe designed by the author and a coworker.⁵⁾

The image of the grating G_1 appears in the plane of G_2 which is the replica of G_1 as Jones' optical lever. G_2 is not split into two halves but slightly tilted to the image of G_1 so that the moiré fringes are observed. As M_1 rotates, the moiré fringes move almost perpendicular to the lines of grating. Two slits are placed behind G_2 so that the intensity of the light through each slit changes with a phase opposite to the other. These light beams fall on the photodiodes D and D' . Since the back focal plane of L_3 is the spectral plane of G_1 , the mirror is placed there so as to reflect only the zeroth and the first order of the diffraction image. The slit S_3 is placed at the back focal plane of L_4 which is the spectral plane of the moiré fringes, and its position is adjusted so that the detectors receive only the diffracted rays of the first order group.

With these arrangements the detector output changes sinusoidally with the rotating angle of M_1 , thus the system can be self-calibrated. This is the feature of the device.

Sensitivity of order 0.1 nrad is obtained with the measuring range of $4.5\mu\text{rad}$. The accuracy is limited by the error of determining the average pitch distance of the gratings.

3 Discrimination of the Direction of Translation

It is essential to provide a method of discriminating the direction when we measure displacement utilizing periodic pattern either natural or artificial, since great amount of measuring error is expected without it when the pattern oscillates around a measuring point.

Let us restrict to the case of pattern counting for simplicity. Suppose we get a pulse whenever the intensity of the pattern reaches the peak at the detecting point as the pattern moves. When the pattern moves in one direction, the displacement is determined by the number of counts multiplied by the pitch of the pattern. When the pattern oscillates around the detecting point, however, the number of counts accumulates as if the pattern is moving in one direction even though the average displacement is zero without the discrimination of the direction.

In order to avoid the error, following logic is introduced.

Two detectors are fixed $1/4$ of the period apart. The configuration of two patterns displaced $1/4$ of the period with one detector is also usable. The output signals of the detectors are shown

in Fig. 13(a) and (b), when the pattern moves to right firstly then turns to left. The analog signals are shaped squared by shaping circuits. The squared signal on the bus(b) is used to open and close the gate circuits, while that on the bus(a) is branched to have two signals of opposite phase. Pulses are obtained by differentiating them and triggering the monostable multivibrators(Fig.13(e) and (f)). When the pattern moves to right, only pulses through the upper gate emerges(Fig.13(h)), and pulses through the lower gate emerges when the pattern moves to left (Fig.13(i)). Introducing these pulses to a reversible counter, exact counts are available even if the pattern moves to-and-fro. Fig.14. shows the block diagram. It is possible to divide the period into one fourth applying this logic as shown diagrammatically in Fig.15.

4. Techniques for Maintaining Accuracy and Precision Relevant to Geometric Metrology

Various techniques are applied in order to maintain the required accuracy and precision of length or angle measurement. They can be divided into two categories: the elimination of systematic errors and the enhancement of signal-to-noise(SN) ratio.

When the direction of translation of the measured object is not aligned to the line of the reference scale, the systematic error occurs unless the direction of the translation is kept perfectly straight which cannot be expected practically. This is called the Abbe's error and must be taken account at the precision measurement of length or displacement.

Fine mechanism to drive the measured object close to parallel translation have been devised. Although almost ideal translation is now able to realize, it is still insufficient for the extreme precision measurement, and the methods compensating the incomplete translation have been pursued.

Change of the refractive index causes the change of optical path length and results in the systematic error when the measurement is performed in air.

Multiple reflection is efficiently utilized for obtaining high sensitivity as well as high SN ratio in the two beam interferometer, since the division of the interference fringes can be made possible in integral order.

Modulation techniques are applied for the analog interpolation as well as for the enhancement of SN ratio.

These techniques are reviewed in this section.

4.1 Fine mechanisms for linear translation and rotation

There are two kinds of philosophy in designing fine mechanism: the kinematic design and the elastic design.

The kinematic design is based on the kinematic principle which states that the number of points of contact and the number of degrees of freedom between two mating parts should always total six. It follows immediately that the number of points of contact must be $6-n$, where n is the given degrees of freedom. Features of the kinematic method are that its part can be made an installed, or replaced, without altering the positional system. Secondary, wear does not induce play in the coupling mode, however it may introduce dimensional

change.

Disadvantages are that there is need for some application of a locator force to hold parts together, such as gravity, magnetism or small springs. The points of contact are also heavily loaded and consequently operate under conditions of high stress.

Fig.16 shows a translating stage designed by the kinematic principle.

The elastic design is based on the opposite philosophy: all components are joined rigidly but allowed to flex within their elastic properties. In cases where only limited translation and rotation is desired, the elastic method is probably the best.

The features of the elastic design are that there are no backlash in relative motion of parts which is desirable for making small displacement. Secondly no wear is expected and lubrication is needless.

The disadvantages are that the driving force for translation depends on the amount of displacement. Hysteresis occurs in the elastic deformation. The parallel spring mechanism as shown in Fig.17 is the typical example. The mechanism shown in Fig.18 has the advantage over the simple parallel spring that the stage moves

parallel to the bases.⁶⁾

In practice the method becomes somewhere in between totally kinematic and elastic, being called semi-kinematic.

Link mechanism has been used for translation but it has been considered not suitable for fine motion because of backlash at the joints. Recent progress in precision machining makes it possible to manufacture a monolithic mechanism which has flexure hinges and works as a link mechanism. The simplest form is shown in Fig.19. This technique is applied to design a goniometer having a point where pure

rotation is realized. Fig.20 is an example.⁷⁾ Another application

finds a reduction of stroke as shown in Fig.21.⁸⁾

4.2 Compensation for the Incomplete Translation

Reflecting mirror is attached to a moving part when its displacement is measured by an optical interferometer. The arrangement as shown in Fig.2 gives rise to error when the mirror rotates around the axis perpendicular to the plane. The beam reflected by the cube-corner prism is parallel to the incident beam regardless of rotation around the axis normal to the incident beam, and it is used instead of the plane mirror. The cat's eye reflector, which consists of a mirror fixed on the focal plane of a lens is also used.

The Fabry-Perot interferometer with flat and confocal mirrors as shown in Fig.4 is insensitive to the incomplete translation, but not free from the Abbe's error.

Polarized light from the laser has been suitably used to set up two beam polarizing interferometer. Fig.22 shows the double-passed Michelson interferometer designed by Bennet for the measurement of

expansion coefficient of the specimen.⁹⁾ The cube-corner reflector

outside the interferometer arms is used to recombine two beams by the polarizing beam splitter. The multiple-path interferometer based on the principle has been designed by Morokuma whose arrangement

is shown in Fig.23.¹⁰⁾ Tanaka has set up very sensitive two beam polarizing interferometer with the prism specially designed so that the optical path length of two beams may be equal in it, thus the change of the refractive index of the prism due to temperature

change does not cause the systematic error.¹¹⁾

4.3 Multiplication of the Optical Path

When the light beam in one path of the Michelson interferometer is reflected many times, the optical path length is multiplied and the sensitivity is also multiplied since the "scale interval" is shortened.

It is realized by replacing two mirrors in Fig.3 by two prisms. Fig.24 illustrates the principle. Various variations have been devised using cube-corner prisms.

4.4 Use of Modulation Techniques

Output drift of the measuring system deteriorates the results of measurement and is very difficult to remove. There are many sources causing the drift, and modulation technique is efficient for removing the drift due to the electronic circuits.

The spectrum of noises in the electronic circuits has the shape

of $f^{-\alpha}$ in low frequency region, α being approximately -1. The SN ratio gets very poor when the spectrum of the signal is localized in the region, and it is very often the case. The signal spectrum can be shifted to higher frequency region by modulating the signal and the SN ratio is considerably improved. The amplitude or the frequency can be modulated: the amplitude modulation and the frequency modulation. The modulated signal is demodulated to recover the original signal.

In demodulating the amplitude-modulated signal, it is necessary to discriminate the polarity of the signal and the phase-sensitive detector (PSD) is used for the purpose. Let v_s and v_r be the input and

reference signal, respectively, and expressed $V_s = V_s \sin(\omega_0 t + \phi)$ and $v_r = V_r \sin \omega_0 t$,

respectively, where ω_0 is the modulating (angular) frequency. The product of v_s and v_r is given

$$v_s v_r = \frac{V_s V_r}{2} [\cos \phi - \cos(2\omega_0 t + \phi)], \quad (7)$$

and we obtain the output signal V_{out}

$$V_{out} = \frac{V_s V_r}{2} \cos \phi, \quad (8)$$

by filtering out the high frequency component.

This is the principle of the phase-sensitive detection: V_{out} is proportional to V_s when the modulated signal is in phase as the

reference, and proportional to $-V_s$ when the phase is opposite to the reference.

In order to demodulate the frequency-modulated signal, it is necessary to discriminate the polarity of the frequency difference and the frequency discriminator is used for the purpose.

4.4.1 Amplitude modulation

The mechanical chopper is used to chop light beam and amplitude modulated signal is obtained. Two vanes driven by a tuning fork are conveniently used. The oscillating frequency as high as 2,000 Hz is available, but the aperture decreases as the frequency.

A vibrating mirror is also used for the amplitude modulation of light beam. Fig.25 shows the photoelectric microscope for detecting the graduation lines of the standard scale as the example.

4.4.2 Phase/frequency Modulation

When the electric field is applied to some kinds of crystal, the refractive index changes proportional to the field strength. This is called the Pockels effect.

Phase modulation of the linearly polarized light beam is possible using this effect, since the retardation in the crystal is changed periodically when a.c. voltage is applied. Use of the Pockels cell in the polarizing interferometer, as shown in Fig.26,⁽²⁾ the optical phase is converted to the electric phase by combining the Pockels cell, the photodetector and PSD.

5 Elimination of Environmental Disturbances

It is desirable to keep environmental conditions unchanged for the precision measurement. The requirements get more and more strict as higher precision is needed, and finally we have to give up the effort to keep them unchanged. It then seems to be wise to devise a measurement system which is immune from the disturbances.

Mechanical vibration and the ambient temperature variation are most troublesome disturbances in geometric metrology. Various techniques have been devised to eliminate the disturbances.

5.1 Vibration Isolation

Measurement system used for the geometric metrology usually consists of mechanical parts fixed or elastically supported on a surface plate. When the surface plate happens to vibrate, elastic elements are excited and exhibit damped vibration. If the frequency of the external vibration coincides with the resonant frequency of any of the elastic elements, the amplitude of vibration grows up and it makes the measurement almost impossible.

Two different kinds of philosophy are applied for vibration isolation: impedance mismatching and low pass filtering.

Vibration is transmitted via ground or floor of building and the power spectrum depends on the mechanism of the generation. For instance, a 4-pole induction motor driving rotary pump for evacuation builds up vibration of 25 Hz if operated by 50Hz line, while the spectrum of ground vibration is widely spread from 0.1Hz to more than 10Hz.

Ground vibration propagates through ground as an elastic wave and

it may be reduced when it propagates through different media where occur the impedance mismatching. Digging a deep pit around a base block on which a flat table is put and filling it dry sand in, ground vibration might be reduced because of the impedance mismatching at the boundary, but it is ineffective unless the width of the pit exceeds the wavelength of the sound wave which is more than 10m for 100Hz.

Use of an anti-vibration table is based on the philosophy of low-pass filtering. Let us investigate it in detail.

5.1.1 the seismo-system of one degree of freedom

A system consisted of a weight supported by a spring with a damper is called the seismo-system and is a simple model of the vibration isolator.

The equation of motion of the weight relative to the supporting floor is

$$m\ddot{x} + 2\alpha(\dot{x} - \dot{y}) + k(x - y) = 0, \quad (9)$$

where m , α and k are the mass of the weight, the damping coefficient of the damper and the spring constant of the spring, respectively. x and y are the displacements of the weight and the floor. The mass of the spring is neglected for simplicity. Denoting the Fourier transform of $x(t)$ and $y(t)$ by $X(\omega)$ and $Y(\omega)$ respectively, the absolute transmittivity $T(\omega)$ is given

$$T(\omega) = \left| \frac{X(\omega)}{Y(\omega)} \right| = \left[\frac{1 + (2\zeta\omega/\omega_0)^2}{\{1 - (\omega/\omega_0)^2\}^2 + (2\zeta\omega/\omega_0)^2} \right]^{\frac{1}{2}}, \quad (10)$$

where $\omega_0 = k/m$, $\zeta = \alpha/\sqrt{km}$ and $\omega = 2\pi f$.

$T(\omega)$ is an index of the effect of the vibration isolation and depends on ζ as shown in Fig.27. It is seen that $T(\omega)$ is always smaller than unity when ω/ω_0 is larger than $\sqrt{2}$ independent of ζ , therefore it is desirable to make ω_0 as small as possible. It is not easy to make f_0 smaller than 2Hz, when the weight is supported by a spring, since the weak spring can not support heavy weight. Feedback technique is applied to have the spring constant apparently zero and is useful for light weight.

5.1.2 Vibration Isolator

Four springs are used to support a table in practice and the degree of freedom of motion of the table gets more than unity.

The pneumatic spring has been used recently instead of the metal spring for high performance vibration isolator.

The spring constant of the pneumatic spring is approximately given

$$k = \frac{\gamma p_i S^2}{V_i} \quad (11)$$

where p_i , V_i and S are the pressure, inner volume and the surface area of the air tank, respectively, and γ is the ratio of the adiabatic to isothermal specific heat (see Fig.28).

The system shown in Fig.29 has two natural frequencies with

damping. It's absolute transmittivity is shown in Fig.30 .

As far as the horizontal vibration is concerned, suspension of a table from the ceiling is efficient for vibration isolation. The natural frequency is roughly proportional to the square root of the length of suspension.

5.1.3. Isolation against Sound Wave Disturbance

Sound wave generated by the ventilating fan sometimes causes vibration of the parts fixed on the measuring table even though it is on the vibration isolator. The energy absorbed by these parts are rather small but the sound frequency may happen to coincide with the natural frequency of a part, then large transient noise is generated in the measurement system. It is ideal to put whole system in a vacuum chamber in order to avoid the sound wave disturbance, but it is not always possible. It has been shown efficient to cover a sensitive part with the material absorbing sound power.

5.2 Temperature Control

Precision measurement of length or angle is usually executed in the temperature controlled room where the room temperature variation is expected to be within 1K or less. This is not necessarily sufficient when high precision is required. Electronic equipments and light source used for the measurement generate heat and result in local temperature gradient in the room. Suppose two elements of 5cm long are separately attached to the table and the thermal expansion coefficient differs 10^{-5} K^{-1} . If the temperature difference between two elements is 0.2K, the length difference is $0.1\mu\text{m}$ which may not be negligible in some case.

Room temperature changes periodically when the automatic control is in operation, and the period depends on the thermal capacity of the room and usually the order of 10 min. Better measurement results may be obtained by turning off the automatic control for air conditioning and letting the room temperature drift gradually.

5.3 Compensation for Environmental Disturbances

Differential method is widely adopted to compensate for environmental variation; difference of output of two identical elements is free from temperature variation if they are set close so that they respond to temperature identically but to input in the opposite way. The integrated differential amplifier has advantage on this point since the circuit components are built very closely each other.

Refractive index of air changes with the atmospheric pressure as well as the ambient temperature and causes the change of optical path length. A number of interferometer arrangements based on the Michelson interferometer have been devised for avoiding the error due to the change of refractive index. It is essential to make the path of the transmitted and reflected beams as close as possible. The arrangement shown in Fig.6 is an example.

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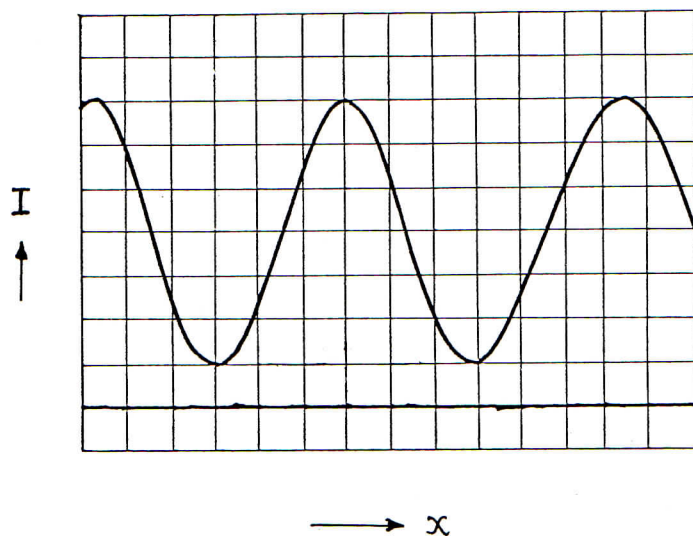


Fig.1 Visibility of the interference fringes

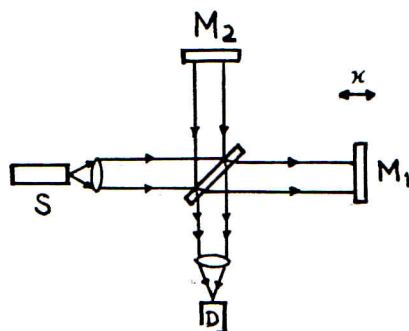


Fig.2 Michelson interferometer

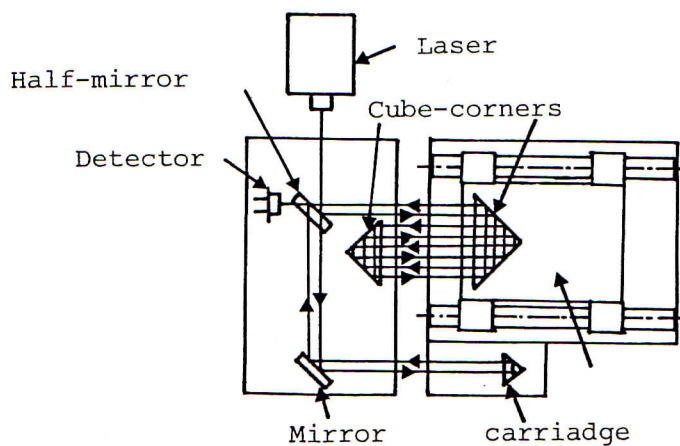


Fig.3 Two-pass Michelson interferometer

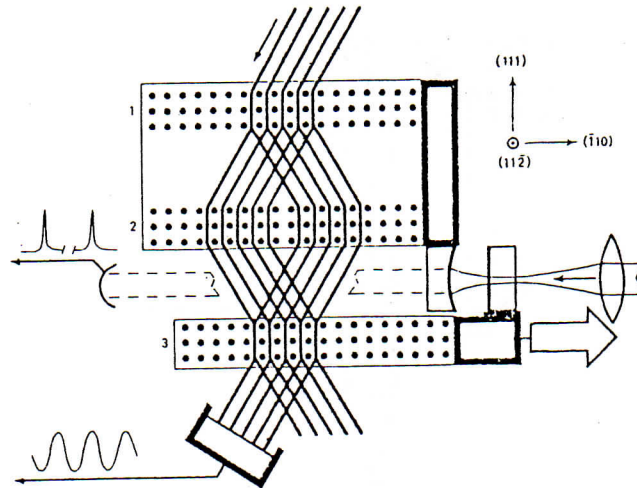


Fig.4 Combined X-ray and optical interferometer

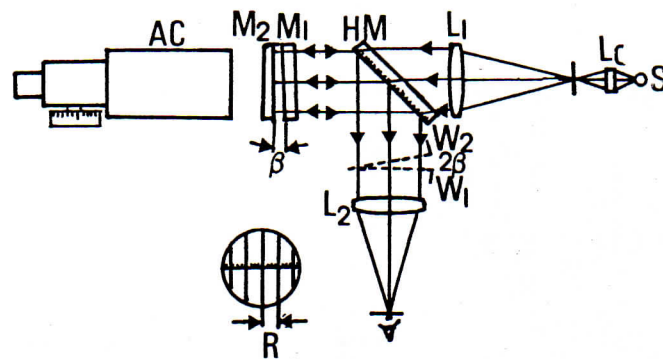


Fig.5 Fizeau interferometer

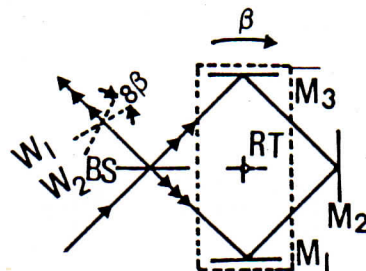


Fig.6 Polygonal path angle interferometer

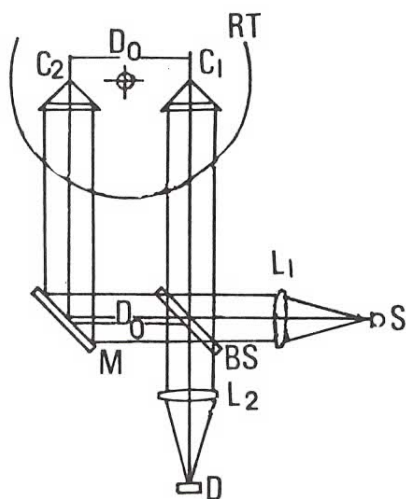


Fig.7 Fringe counting angle interferometer

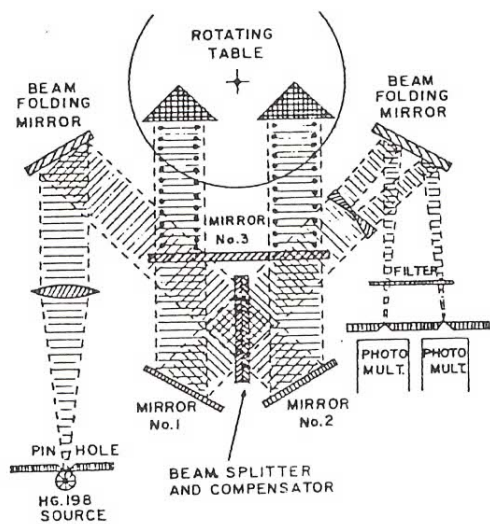


Fig.8 Doubling sensitivity of angle interferometer

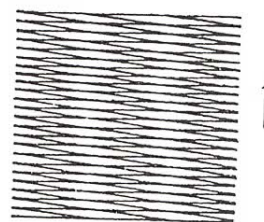


Fig.9 Moire fringe



Fig.10 Electromagnetic scale

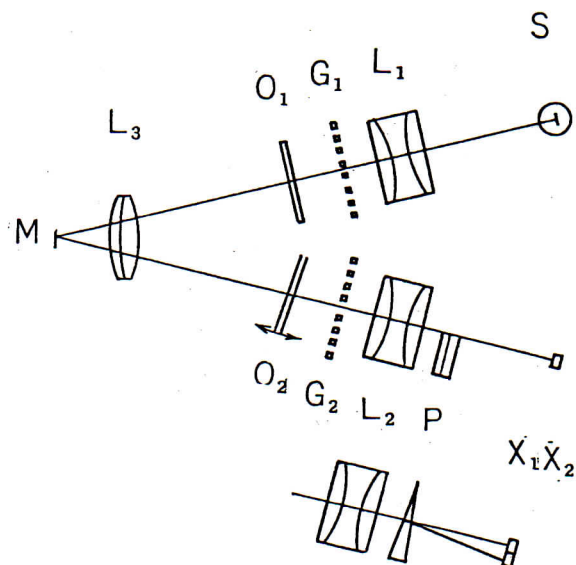


Fig.11 Photoelectric optical lever

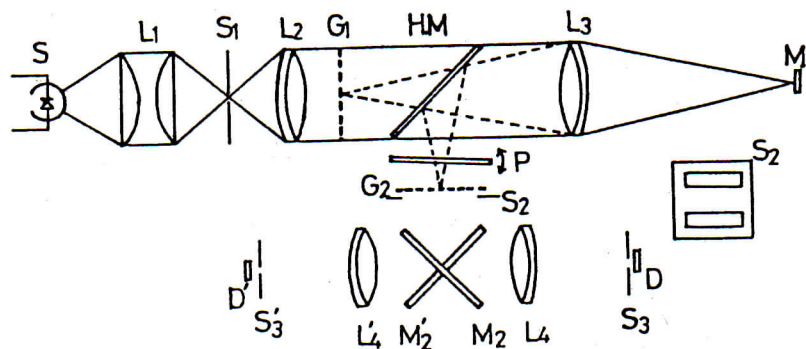


fig.12 Small-angle measuring device

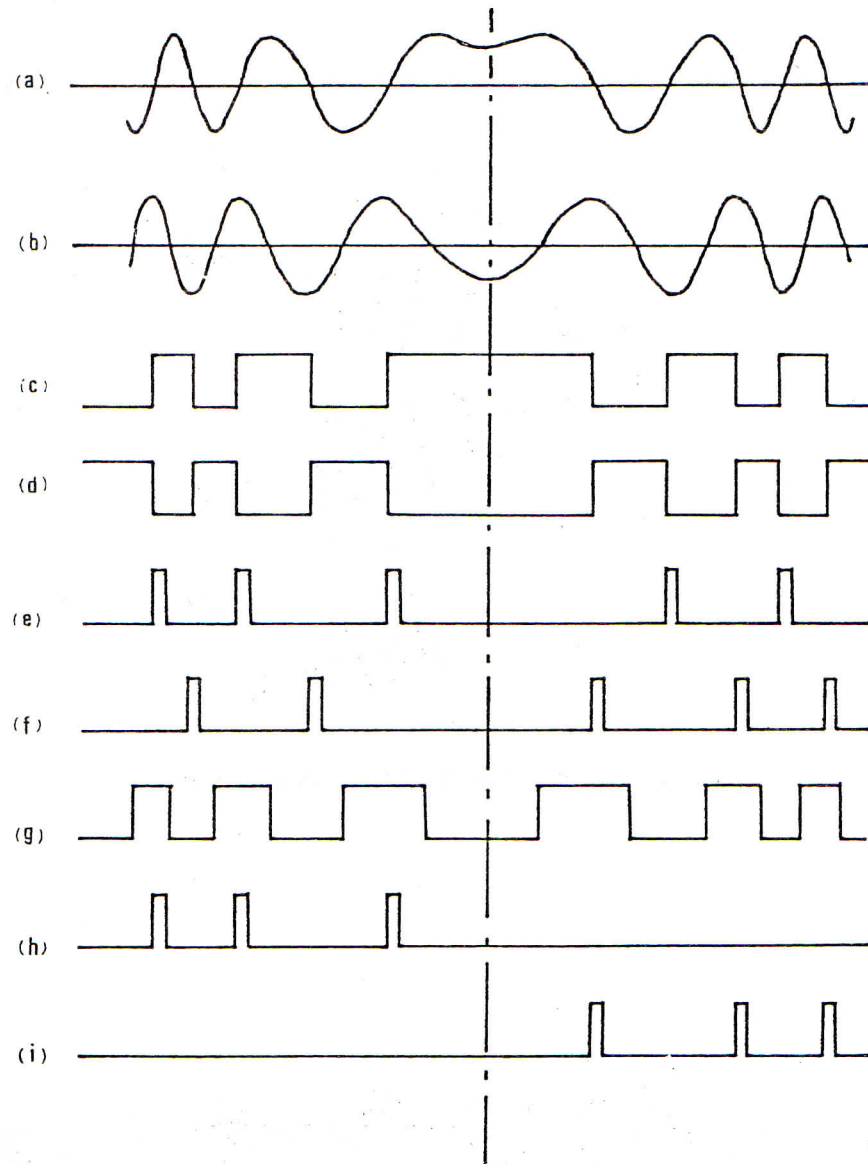


Fig.13 Discrimination of the direction of translation

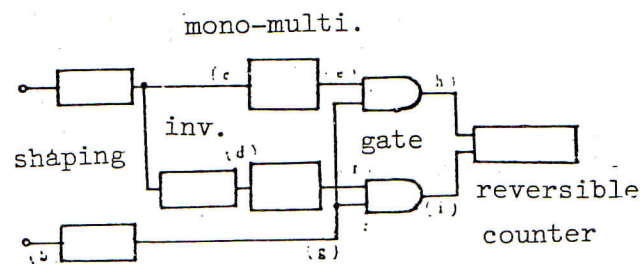


Fig.14 Blockdiagram of the direction discrimination

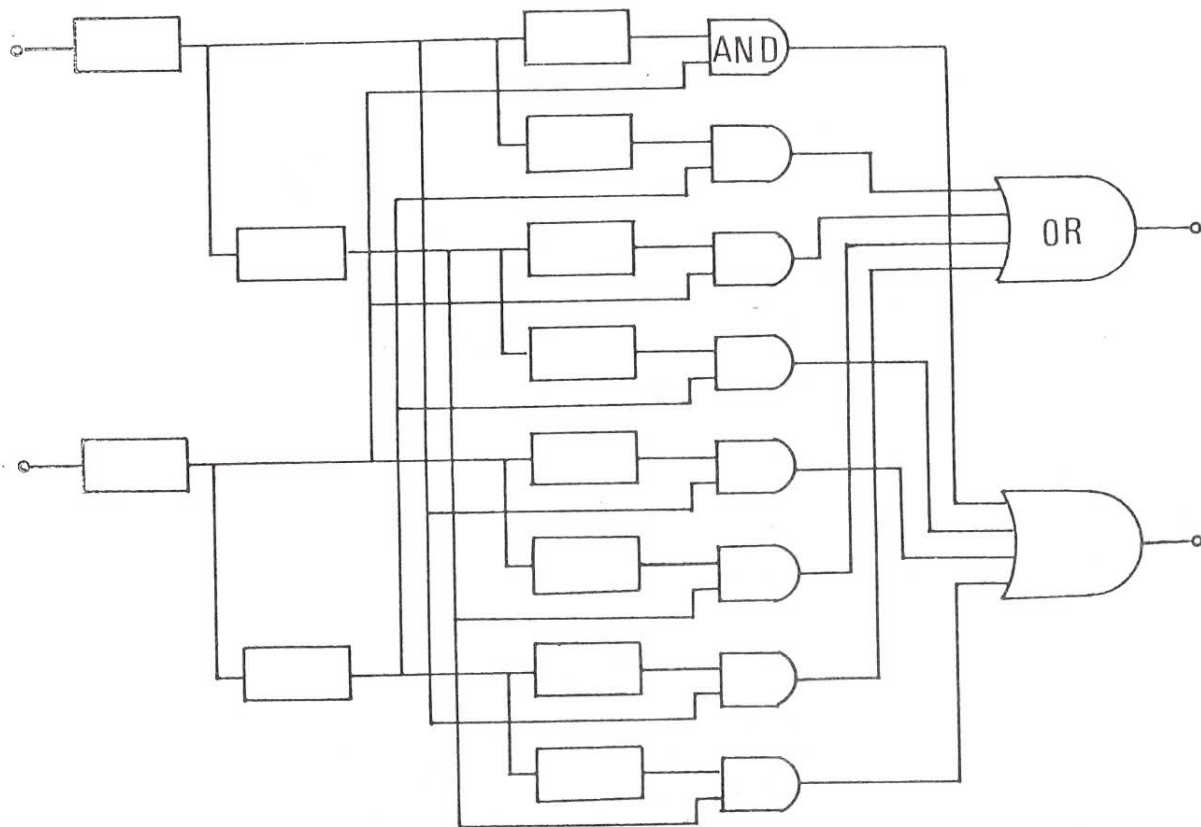


Fig.15 Digital interpolation with the direction discrimination

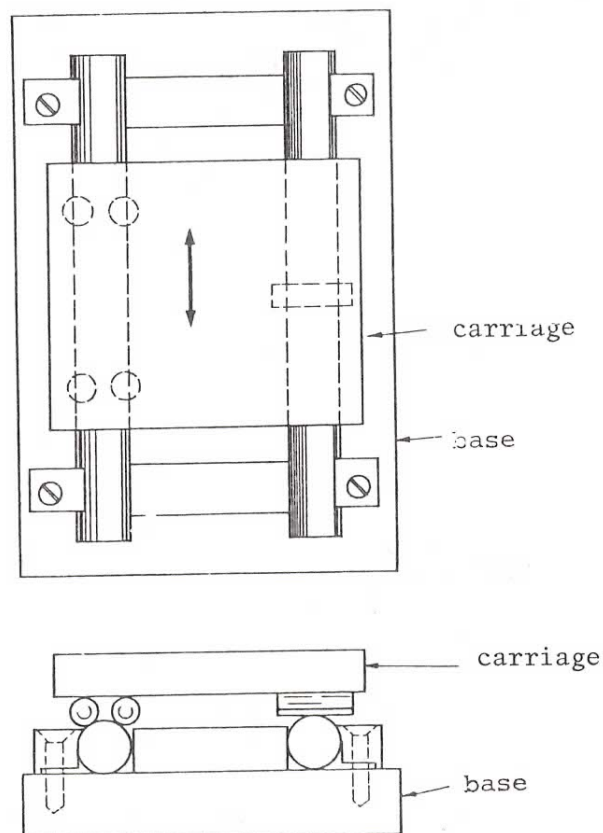


Fig.15 Translation stage

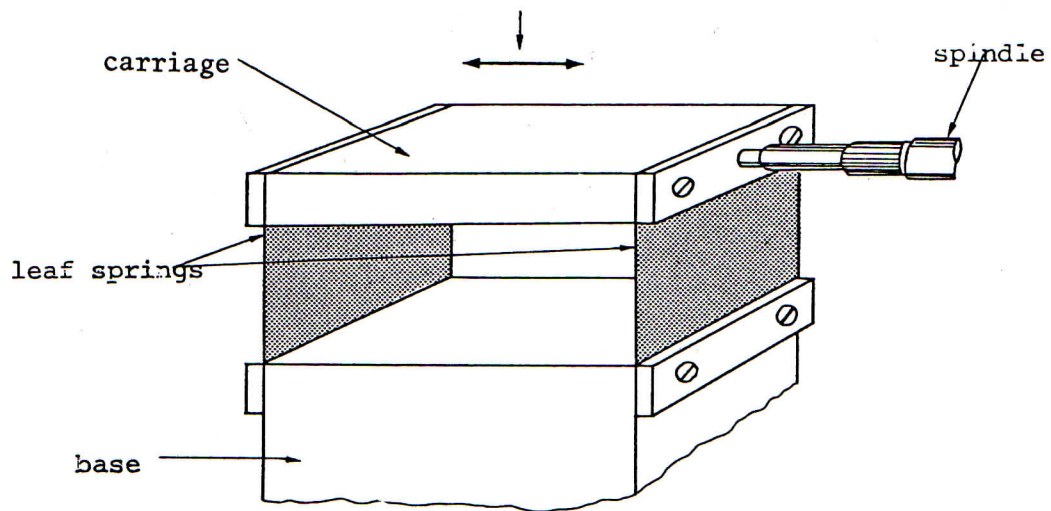


Fig. 17 Parallel spring

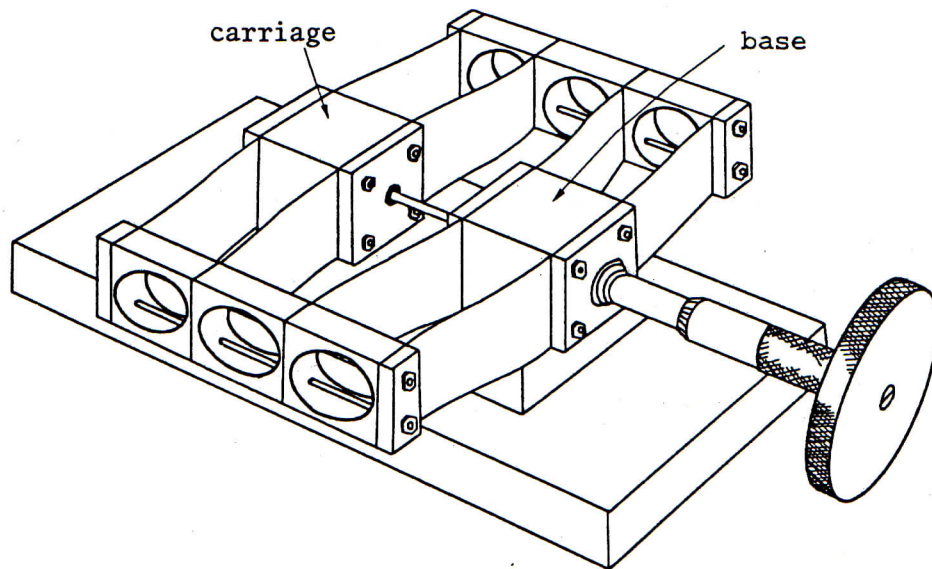


Fig.18 Composite parallel spring

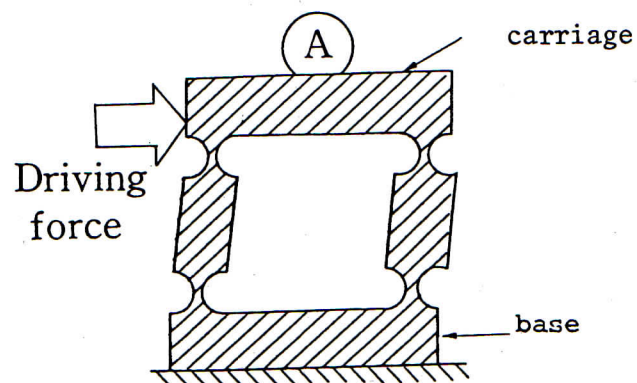


Fig.19 Monolithic parallel spring

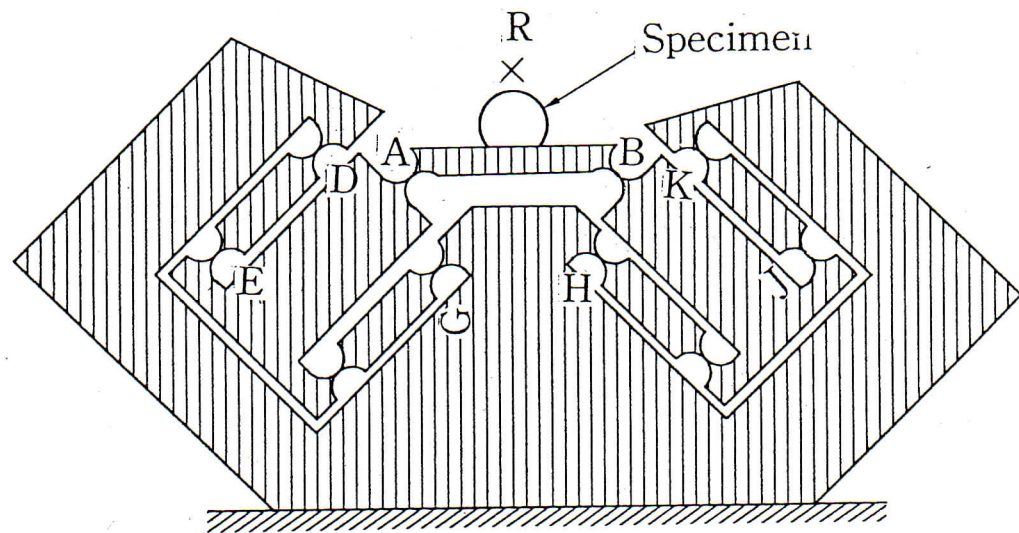


Fig.20 Goniometer with the center of rotation

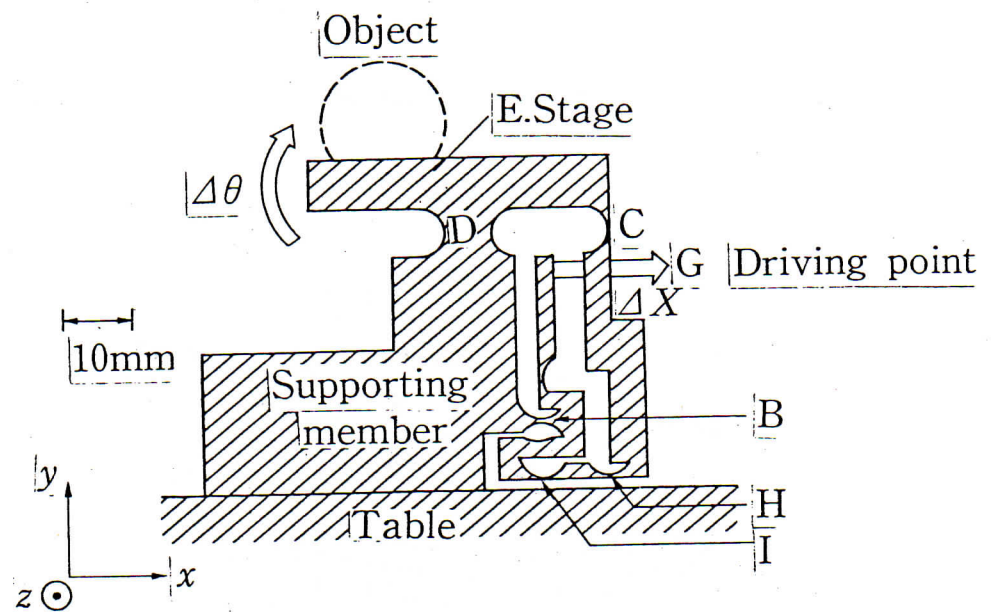


Fig.21 Reduction of stroke

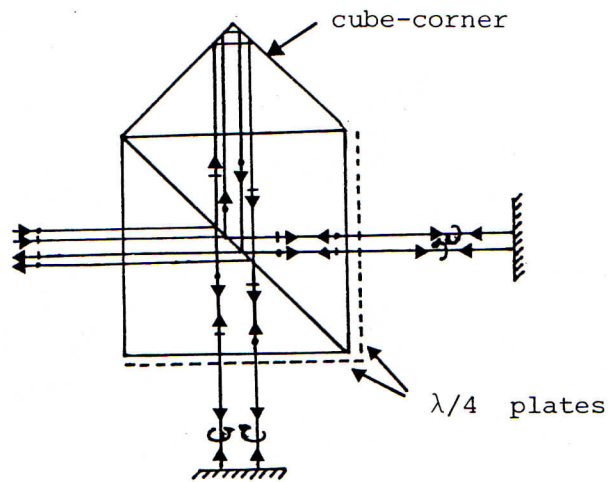


Fig.22 Double-path Michelson interferometer

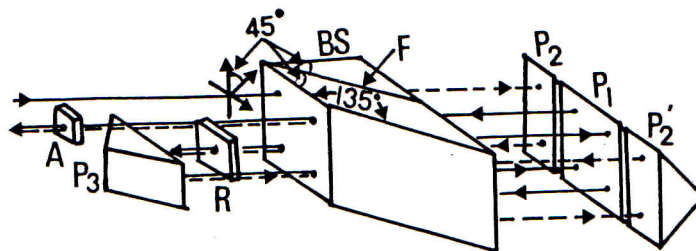


Fig.23 Multiple-path Michelson interferometer

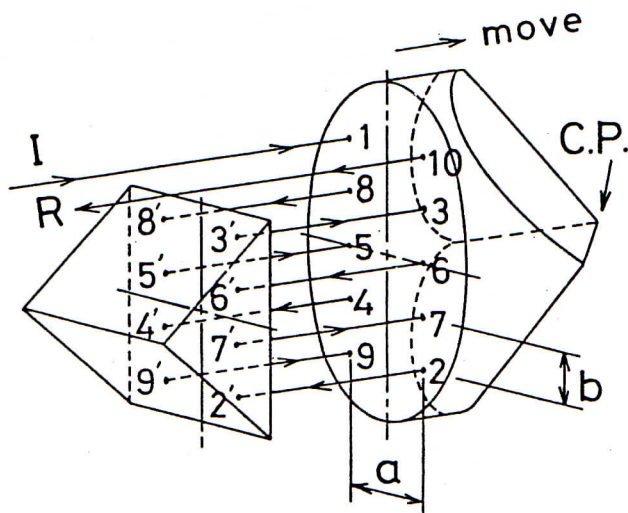


Fig.24 Multiple reflection using the cube-corner prism

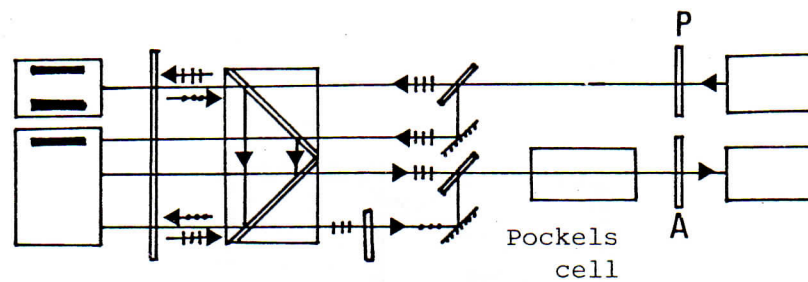


Fig.26 Modulation using the Pockels cell

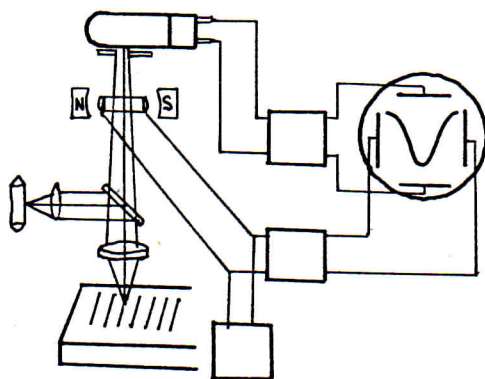


Fig.25 Photoelectric microscope

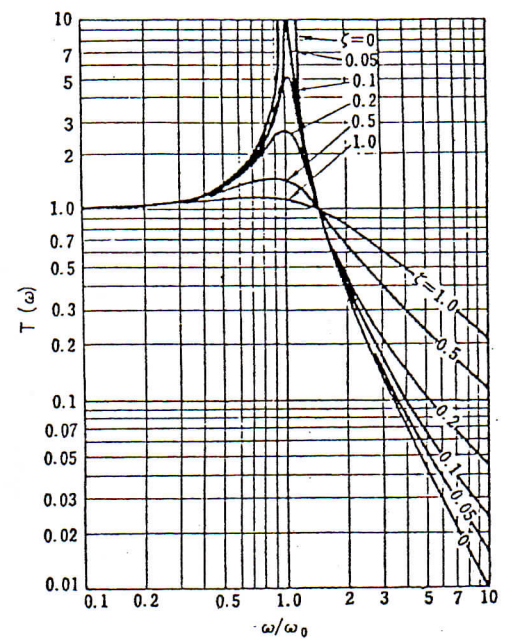


Fig.27 Absolute transmittivity of the system of 1 degree of freedom

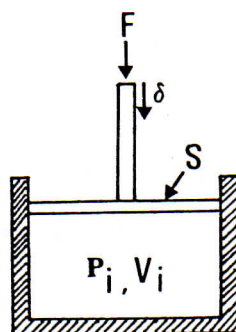


Fig.28 Pneumatic spring

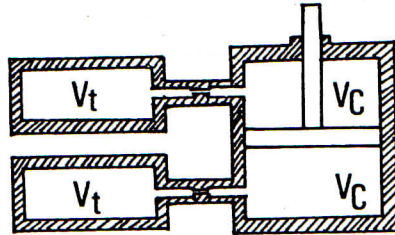


Fig.29 Pneumatic spring with surge tanks

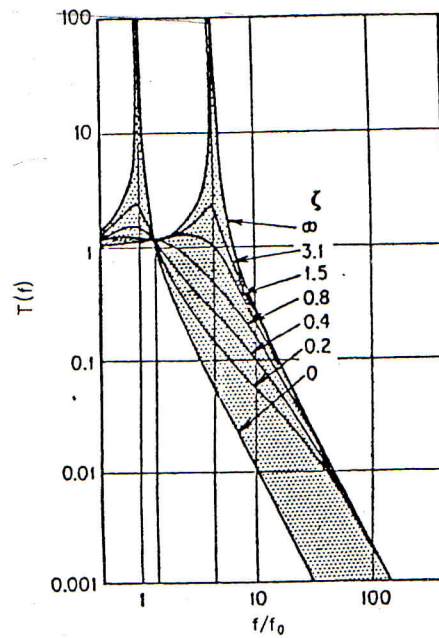


Fig.30 Absolute transmittivity of the vibration isolator with the pneumatic spring